Quantum Mechanics

Linear momentum

Problem 1.- Find one eigen function of the momentum operator $\hat{p} = -i\hbar \frac{d}{dx}$

Solution: To get an eigen function we need to satisfy the equation:

$$-i\hbar \frac{d}{dx}\psi = p\psi$$

where p is a constant (the eigenvalue). Notice that this is a first order differential equation with separable variables:

$$\frac{d\psi}{\psi} = i\frac{p}{\hbar}dx$$

And integrating both sides we get:

$$\int \frac{d\psi}{\psi} = i\frac{p}{\hbar}\int dx \rightarrow \ln\psi = i\frac{p}{\hbar}x + C$$

where C is a constant. The wave function is: $\psi = Ae^{ipx/\hbar}$

Problem 2.- If you measure the momentum of a particle with wave function

$$\psi(x) = \sin kx$$

What values are possible?

Solution: The wave function can be written as the sum of two eigen functions of the momentum operator as:

$$\psi(x) = \sin kx = \frac{e^{ikx} - e^{-ikx}}{2i}$$

The two possible outcomes of a measurement would be $p = \hbar k$ or $p = -\hbar k$.

Problem 3.- The wave function of a particle is given by

$$\psi = e^{i(ax-\omega_1 t)} + e^{i(2ax-\omega_2 t)}$$

Calculate the possible outcomes of a measurement of the momentum.

Solution: Notice that the wave function is a linear combination of eigen functions of the momentum operator, so the only possible outcomes of an experiment are either:

$$p = a\hbar$$

or

$$p = 2a\hbar$$