Quantum Mechanics

Normalization

Problem 1.- The functions ψ_n are orthonormal. Find the value of “a” that will normalize this wave function:

$$\psi = a\psi_1 + \frac{1}{\sqrt{2}}\psi_2 + \frac{1}{2}\psi_3$$

Solution: To normalize the eigen function:

$$\int_{-\infty}^{\infty} \psi^* \psi dx = 1$$

$$\int_{-\infty}^{\infty} \left(a\psi_1 + \frac{1}{\sqrt{2}}\psi_2 + \frac{1}{2}\psi_3 \right)^* \left(a\psi_1 + \frac{1}{\sqrt{2}}\psi_2 + \frac{1}{2}\psi_3 \right) dx = 1$$

There are nine products after multiplying the two terms in the integral, but six of them are cross products and they will give zero when integrating due to orthogonality. Only three terms are left:

$$\int_{-\infty}^{\infty} |a|^2 |\psi_1|^2 + \frac{1}{2}|\psi_2|^2 + \frac{1}{4}|\psi_3|^2 dx = 1$$

But the functions ψ_n are normal, so their integrals are equal to 1:

$$|a|^2 + \frac{1}{2} + \frac{1}{4} = 1 \rightarrow |a|^2 = \frac{1}{4} \rightarrow a = \pm 0.5$$

a can also be a complex number with modulus 0.5

Problem 1a.- The functions ψ_n are orthonormal. Find the value of “a” that will normalize the wave function:

$$\psi = a\psi_1 - a\psi_2 + \frac{1}{2}\psi_3$$

Problem 2.- The functions $\psi_n(x)$ are eigen functions of the problem of a particle of mass “m” in a square well potential of length “a”. You are given the solution at time t=0:

$$\psi(x,0) = A(\psi_1(x) - \psi_2(x) + \psi_3(x))$$

Normalize the wave function and find the solution for time “t”.