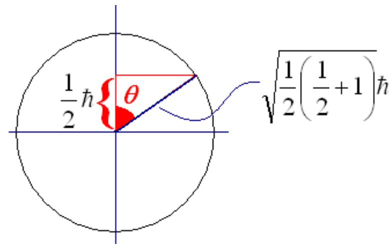
Quantum Mechanics

Angular momentum

Problem 1.- What is the angle between the z-axis and the spin of an electron in the "spin up" state?

Solution: A diagram helps to solve this problem



Notice that the magnitude of the spin is: $\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar$ and the projection on the z-axis is only $\frac{1}{2}\hbar$, so the cosine of the angle is:

$$\cos\theta = \frac{\frac{1}{2}\hbar}{\sqrt{\frac{1}{2}\left(\frac{1}{2}+1\right)}\hbar} = \frac{1}{\sqrt{3}} \rightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = \mathbf{54.7^o}$$

Problem 2.- A particle is prepared in the state: $\psi = \frac{2Y_2^1 + Y_3^2 + Y_3^0}{\sqrt{6}}$. An experiment to measure angular momentum yields $l = 2$. If we measure the magnetic quantum number immediately after, what value do you expect to get?

Solution: When the angular momentum is measured, the wave function will "collapse" to Y_2^1, which has magnetic quantum number **m=1**.

Problem 3.- What are the places where the probability of finding the electron of the hydrogen atom with quantum numbers n = 3, l = 0 and m = 0 vanishes?

Solution: The angular wave function with l = 0 and m = 0 is a constant, so the only places where the probability vanishes are where the radial wave function is zero.

This happens when $1 - \frac{2}{3}\frac{r}{a} + \frac{2}{27}\left(\frac{r}{a}\right)^2 = 0$

The two solutions are **r = 7.1a** and **r = 1.9a**

Problem 4.- What is the angular eigen function that has L^2 eigenvalue of $12\hbar^2$ and L_z eigenvalue of $-2\hbar$?

Solution: If L^2 is $12\hbar^2$ it means that l = 3. The fact that L_z is $-2\hbar$ means that m=-2, so the angular wave function is Y_3^{-2}

Problem 4a.- What is the angular eigen function that has L^2 eigenvalue of $30\hbar^2$ and L_z eigenvalue of $-3\hbar$?

Solution: Notice that $l(l+1) = 30 = 5\times6$, so the value of l is 5. The value of m is -3, which specifies the spherical harmonic Y_5^{-3}