# Quantum Mechanics

## Bohr model

**Problem 1.-** What is the minimum energy necessary to remove the last electron from the ion $C^{+5}$?

**Solution:** The ion $C^{+5}$ will behave like a hydrogenic atom, so to remove the last electron we will need:

$$E = Z^2 \frac{13.6eV}{1^2} = 6^2 \frac{13.6eV}{1^2} = \mathbf{489.6eV}$$

**Problem 2.-** Calculate the energy of the photon emitted if a positronium atom makes a transition from the state with n = 5 to a state with n = 1.

**Solution:** Positronium is another example of a hydrogenic atom, but the reduced mass is half the mass of the electron, so the eigen energies are one half of the hydrogen energies:

$$\Delta E = -\frac{13.6eV}{2}\left(\frac{1}{5^2} - \frac{1}{1^2}\right) = \mathbf{6.53eV}$$

In joules this is: 6.53eV($1.6\times10^{-19}$J/eV) = $1.04\times10^{-18}$J

**Problem 3.-** The energy required to remove both electrons from the helium atom in its ground state is 79.0 eV. How much energy is required to ionize the neutral helium atom (i.e., to remove just the first electron)?

**Solution:** Calculating the energy required to remove the second electron from a helium atom is fairly easy: The ion behaves like a hydrogenic atom, so the energy is given by:

$$E = Z^2 \frac{13.6eV}{1^2} = 2^2 \frac{13.6eV}{1^2} = 54.4eV$$

Since the total energy to remove both electrons is 79.0 eV, to remove the first one we need 79.0-54.4= **24.6eV**

**Problem 4.-** In the hydrogen spectrum, calculate the wavelength for the Lyman radiation n = 4 to n = 1.

**Solution:** The energy of this transition is:

$$\Delta E = -13.6eV\left(\frac{1}{4^2} - \frac{1}{1^2}\right) = -13.6eV\left(\frac{1}{16} - 1\right) = 12.75eV$$

Which in joules is:

$$\Delta E = 12.75eV\left(\frac{1.6\times10^{-19}J}{1eV}\right) = 2.04\times10^{-18}J$$

To get the wavelength, recall that $\Delta E = \frac{hc}{\lambda} \rightarrow \lambda = \frac{hc}{\Delta E}$ and with the values of the problem:

$$\lambda = \frac{(6.62\times10^{-34}\,Js)(3\times10^{8}\,m/s)c}{2.04\times10^{-18}\,J}$$ =**97.4nm**

**Problem 5.-** Given that the binding energy of the hydrogen ground state is $E_o$=13.6eV, the binding energy of the n=3 state of positronium (electron-positron "atom") is:

(A) $E_o/9$ (B) $E_o/18$ (C) $E_o/36$ (D) $2E_o/9$ (E) $4E_o/9$

**Solution:** Recall that the correction to the energies in hydrogenic atoms due to the finite mass of the nucleus is given by multiplying by the ratio of reduced mass to electron mass:

$$E_{corrected} = \frac{-E_o}{n^2}\frac{\mu}{m}$$

Where $E_o$ is -13.6eV, $\mu$ is the reduced mass of the electron-nucleus pair and *m* is the mass of the electron. This is a small correction in the case of hydrogen and deuterium, but it is ½ in the case of the positronium atom, since the mass of the positron is equal to the mass of the electron. So:

$$E_{Positronium} = \frac{-E_o}{2n^2} = \frac{-E_o}{18}$$

**Answer: B**

**Problem 6.-** Calculate the wavelength for the Lyman radiation n = 4 to n = 1 for hydrogen and deuterium considering the finite mass of the nucleus. What is the difference between the two wavelengths?

**Solution:** Recall that the correction to the energies in hydrogenic atoms due to the finite mass of the nucleus is given by multiplying by the ratio of reduced mass to electron mass:

$$\Delta E_{Hydrogen} = E_o\left(\frac{1}{1^2} - \frac{1}{4^2}\right)\frac{m_{proton}}{m_e + m_{proton}}$$

$$\Delta E_{Deuterium} = E_o\left(\frac{1}{1^2} - \frac{1}{4^2}\right)\frac{m_{deuteron}}{m_e + m_{deuteron}}$$

The wavelengths are calculated using $\lambda = \frac{hc}{\Delta E}$ L:

$$\lambda_{Hydrogen} = \frac{hc}{E_o\left(\frac{1}{1^2}-\frac{1}{4^2}\right)\frac{m_{proton}}{m_e+m_{proton}}} = \frac{16hc}{15E_o}\left(\frac{m_e+m_{proton}}{m_{proton}}\right)$$

$$\lambda_{Deuterium} = \frac{hc}{E_o\left(\frac{1}{1^2}-\frac{1}{4^2}\right)\frac{m_e m_{deuteron}}{m_e+m_{deuteron}}} = \frac{16hc}{15E_o}\left(\frac{m_e+m_{deuteron}}{m_{deuteron}}\right)$$

Using the value of $E_o = 2.17990\times10^{-18}$ J =13.6eV and the masses of electron, proton and deuteron:

$$\lambda_{Hydrogen} = 97.2535nm$$
$$\lambda_{Deuterium} = 97.2270nm$$