Quantum Mechanics

Bosons and Fermions

Problem 1.- Classify the following neutral atoms as fermions or bosons:

$^{14}_{7}\text{N}$ ___________

$^{23}_{11}\text{Na}$ ___________

$^{93}_{41}\text{Nb}$ ___________

$^{259}_{102}\text{No}$ ___________

Solution: We need to count the number of fermions (electrons, protons and neutrons) if the number is even, the neutral atom is a boson, otherwise a fermion:

$^{14}_{7}\text{N}$ **Fermion**

$^{23}_{11}\text{Na}$ **Boson**

$^{93}_{41}\text{Nb}$ **Boson**

$^{259}_{102}\text{No}$ **Fermion**

Problem 2.- Write down a symmetric wave function of three identical bosons, using the wave functions ψ_1, ψ_2 and ψ_3.

Solution: If we sum all the possible combinations of the three wave functions we will get a symmetric wave function with parity +1:

$$\psi_1(\vec{r}_1)\psi_2(\vec{r}_2)\psi_3(\vec{r}_3)+\psi_1(\vec{r}_1)\psi_3(\vec{r}_2)\psi_2(\vec{r}_3)+\psi_2(\vec{r}_1)\psi_1(\vec{r}_2)\psi_3(\vec{r}_3)+\psi_2(\vec{r}_1)\psi_3(\vec{r}_2)\psi_1(\vec{r}_3)+$$
$$\psi_3(\vec{r}_1)\psi_1(\vec{r}_2)\psi_2(\vec{r}_3)+\psi_3(\vec{r}_1)\psi_2(\vec{r}_2)\psi_1(\vec{r}_3)$$

To normalize the wave function, we need to divide it by $\sqrt{6}$.

Problem 3.- Write down an antisymmetric wave function of three identical fermions, using the wave functions ψ_1, ψ_2 and ψ_3.

Solution: We could generate a Slater determinant and write down an antisymmetric wave function:

$$\begin{vmatrix} \psi_1(\vec{r}_1) & \psi_1(\vec{r}_2) & \psi_1(\vec{r}_3) \\ \psi_2(\vec{r}_1) & \psi_2(\vec{r}_2) & \psi_2(\vec{r}_3) \\ \psi_3(\vec{r}_1) & \psi_3(\vec{r}_2) & \psi_3(\vec{r}_3) \end{vmatrix}$$

To normalize the wave function, we need to divide it by $\sqrt{6}$.

Problem 4.- A system containing two identical particles is described by a wave function of the form:

$$\psi = \frac{1}{\sqrt{2}}\left[\psi_a(x_1)\psi_b(x_2) - \psi_a(x_2)\psi_b(x_1)\right]$$

Where x_1 and x_2 represent the spatial coordinates of the particles and a and b represent all the quantum numbers, including spin, of the states that they occupy. The particles might be:

(A) Deuterons.

(B) Neutral sodium atoms ${}^{23}_{11}Na$

(C) Alpha particles

(D) Neutral nitrogen atoms ${}^{14}_{7}N$

(E) Neutral rubidium-85 isotopes ${}^{85}_{37}Rb$

Solution: The wave function given has parity -1, that is, the wave function changes sign when two particles are swapped:

$$\psi = \frac{1}{\sqrt{2}}\left[\psi_a(x_1)\psi_b(x_2) - \psi_a(x_2)\psi_b(x_1)\right]$$

This means that it represents fermions.
The only fermions in the list are neutral nitrogen atoms ${}^{14}_{7}N$.

Answer: D