Quantum Mechanics

Metals

Problem 1.- Find the density of valence electrons in gold (in m^{-3}), knowing that its valence is 1, its density is $19.3\times10^3 kg/m^3$ and its atomic mass number is 197.

Solution: To find the density of valence electrons, notice that one mol of gold occupies a volume equal to:

$$V_{mol} = \frac{0.197kg}{19.3\times10^3 kg/m^3} = 1.021\times10^{-5} m^3$$

This volume contains one Avogadro's number of valence electrons, so the density of electrons is:

$$n = \frac{N}{V} = \frac{6.025\times10^{23}}{1.021\times10^{-5} m^3} = \mathbf{5.9\times10^{28}/m^3}$$

Problem 2.- Find the Fermi level of gold (in eV), knowing that its valence is 1, its density is $19.3\times10^3 kg/m^3$ and its atomic mass number is 197.

Solution: Knowing the density of electrons, we can now calculate the Fermi energy:

$$E_{Fermi} = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3} = \frac{(1.05\times10^{-34}Js)^2}{2(9.1\times10^{-31}kg)}\left(3(3.1416)^2(5.9\times10^{28}/m^3)\right)^{2/3} = 8.79\times10^{-19}\text{J}$$

This value in eV is:

$$E_{Fermi} = 8.79\times10^{-19}\text{J}/(1.6\times10^{-19}\text{J/eV}) = \mathbf{5.5\ eV}$$

Problem 3.- Find the Fermi velocity of electrons in gold (The speed of the electrons with Fermi energy), knowing that its valence is 1, its density is $19.3\times10^3 kg/m^3$ and its atomic mass number is 197.

Solution: To find the Fermi velocity, notice that the Fermi energy is the kinetic energy of the electrons at the Fermi surface, so

$$8.79\times10^{-19}\text{J} = \frac{1}{2}mv^2 \rightarrow v_{Fermi} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(8.79\times10^{-19}J)}{9.1\times10^{-31}kg}} = \mathbf{1.39\times10^6\ m/s}$$

Problem 4.- Find the "Fermi temperature" of silver. That is, find the Fermi energy and convert the result to kelvin dividing by Boltzmann constant. The valence of silver is 1, its density is 10.5×10^3 kg/m^3 and its atomic mass is 107.9 u.

Solution: First let us find the density of valence electrons, notice that one mol of silver occupies a volume equal to:

$$V_{mol} = \frac{0.1079kg}{10.5\times10^3 kg/m^3} = 1.027\times10^{-5} m^3$$

This volume contains one Avogadro's number of valence electrons, so the density of electrons is:

$$n = \frac{N}{V} = \frac{6.025\times10^{23}}{1.027\times10^{-5} m^3} = 5.863\times10^{28}/\text{m}^3$$

Knowing the density of electrons, we can now calculate the Fermi energy:

$$E_{Fermi} = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3} = \frac{(1.05\times10^{-34} Js)^2}{2(9.1\times10^{-31} kg)}\left(3(3.1416)^2(5.863\times10^{28}/m^3)\right)^{2/3} = 8.75\times10^{-19}\text{J}$$

This value in kelvin is:

$E_{Fermi} = 8.79\times10^{-19}\text{J}/(1.38\times10^{-23}\text{J/K})$ = **63,400 K**