Quantum Mechanics

Schrödinger Equation

The Schrödinger equation in three dimensions for a time dependent wave function is

$$i\hbar\frac{\partial}{\partial t}\Psi\left(t,\vec{r}\right)=-\frac{\hbar^2}{2m}\nabla^2\Psi\left(t,\vec{r}\right)+V(\vec{r})\Psi\left(t,\vec{r}\right)$$

Where V is the potential and $\Psi\left(t,\vec{r}\right)$ is the wave function, which depends on time and position. The interpretation is that the square of its modulus (Ψ can be complex) is the probability density, meaning that we can multiply it by a volume and get the probability of finding the particle in that volume at that time.

$$\text{Probability}=\left|\Psi\left(t,\vec{r}\right)\right|^2 volume$$

If we can separate the time dependence from the wave function, we can simplify the equation as follows:

We define $\Psi\left(t,\vec{r}\right)=\psi(\vec{r})e^{-i\frac{E}{\hbar}t}$,

so, the first term in the equation becomes $i\hbar\frac{\partial}{\partial t}\Psi\left(t,\vec{r}\right)=-i\frac{E}{\hbar}\psi(\vec{r})e^{-i\frac{E}{\hbar}t}$ and replacing this in the original equation we get

$$i\hbar\left(-i\frac{E}{\hbar}\right)\psi(\vec{r})e^{-i\frac{E}{\hbar}t}=-\frac{\hbar^2}{2m}\nabla^2\psi(\vec{r})e^{-i\frac{E}{\hbar}t}+V(\vec{r})\psi(\vec{r})e^{-i\frac{E}{\hbar}t}$$, which gives us:

$$-\frac{\hbar^2}{2m}\nabla^2\psi+V(\vec{r})\psi=E\psi$$

Which is the time-independent Schrödinger equation.