Quantum Mechanics

Particle in a box

The time-independent Schrödinger equation in one dimension is:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V(x)\psi = E\psi$$

Where the full wave function is $\Psi(t,x) = \psi(x)e^{-i\frac{E}{\hbar}t}$

One of the simplest potentials that we can have is a box, defined as:

$$V(x) = \begin{cases} 0 \; if \;\; 0 < x < a \\ \infty \;\; otherwise \end{cases}$$

In that case the solutions are:

$$\psi(x) = \sqrt{\frac{2}{a}}\sin\left(n\frac{\pi}{a}x\right)$$

Where n can be any positive integer. The kinetic energy for this state is:

$$E = n^2\frac{\pi^2\hbar^2}{2ma^2}$$

You can also write this energy as $E = n^2\dfrac{h^2}{8ma^2}$