Quantum Mechanics

Plane Waves

The time-independent Schrödinger equation in three dimensions is:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(\vec{r})\psi = E\psi$$

Where the full wave function is $\Psi(t,\vec{r}) = \psi(\vec{r})e^{-i\frac{E}{\hbar}t}$

The simplest possibility is V=0, which corresponds to free space. Then the equation reduces to

$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi$$

One possible solution is the plane wave $\psi = e^{i\vec{k}\cdot\vec{r}}$, which once replaced in the equation gives us

$$-\frac{\hbar^2}{2m}\nabla^2 e^{i\vec{k}\cdot\vec{r}} = Ee^{i\vec{k}\cdot\vec{r}} \rightarrow E = \frac{\hbar^2}{2m}k^2$$

Since k can have any value, this is sometimes called the continuum. In addition, if we apply the momentum operator $p = -i\hbar\nabla$, we have a neat interpretation of k.

$$\vec{p} = \langle\psi|\vec{p}|\psi\rangle = e^{-i\vec{k}\cdot\vec{r}}\left(-i\hbar\nabla\right)e^{i\vec{k}\cdot\vec{r}} = \hbar\vec{k}$$

You could also infer this from the energy equation that derives from the usual $E = \frac{p^2}{2m}$