Quantum Mechanics

Simple Harmonic Oscillator

The time-independent Schrödinger equation in one dimension is:

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi + V(x)\psi = E\psi$$

Where the full wave function is $\Psi\left(t,x\right) = \psi(x)e^{-i\frac{E}{\hbar}t}$

One of the simplest potentials that we can have is a simple harmonic oscillator

$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2$$

In that case, the solutions are:

$$\psi(x) = \sqrt[4]{\frac{m\omega}{\pi\hbar}}\frac{1}{\sqrt{2^n n!}}H_n\left(u\right)e^{-u^2/2}$$

Where $u = \sqrt{\frac{m\omega}{\hbar}}x$ and H_n is the Hermite polynomial.

The energy for this state is:

$$E = (n+1/2)\hbar\omega$$

List of Hermite Polynomials

$H_0 = 1$

$H_1 = 2u$

$H_2 = 4u^2 - 2$

$H_3 = 8u^3 - 12u$

$H_4 = 16u^4 - 48u^2 + 12$

$H_5 = 32u^5 - 160u^3 + 120u$

$H_6 = 64u^6 - 480u^4 + 720u^2 - 120$

$H_7 = 128u^7 - 1344u^5 + 3360u^3 - 1680u$

$H_8 = 256u^8 - 3584u^6 + 13440u^4 - 13440u^2 + 1680$

$H_9 = 512u^9 - 9216u^7 + 48384u^5 - 80640u^3 + 30240u$

$H_{10} = 1024u^{10} - 23040u^8 + 161280u^6 - 403200u^4 + 302400u^2 - 30240$

$H_{11} = 2048u^{11} - 56320u^9 + 506880u^7 - 1774080u^5 + 2217600u^3 - 665280u$