

Quantum Mechanics

Table of Associated Legendre Functions

$$P_0^0(x) = 1$$

$$P_1^0(x) = x$$

$$P_1^1(x) = (1-x^2)^{1/2}$$

$$P_2^0(x) = \frac{1}{2}(3x^2 - 1)$$

$$P_2^1(x) = 3x(1-x^2)^{1/2}$$

$$P_2^2(x) = 3(1-x^2)$$

$$P_3^0(x) = \frac{1}{2}(5x^3 - 3x)$$

$$P_3^1(x) = \frac{3}{2}(5x^2 - 1)(1-x^2)^{1/2}$$

$$P_3^2(x) = 15x(1-x^2)$$

$$P_3^3(x) = 15(1-x^2)^{3/2}$$

$$P_4^0(x) = \frac{1}{8}(35x^4 - 30x^2 + 3)$$

$$P_4^1(x) = \frac{5}{2}(7x^3 - 3x)(1-x^2)^{1/2}$$

$$P_4^2(x) = \frac{15}{2}(7x^2 - 1)(1-x^2)$$

$$P_4^3(x) = 105x(1-x^2)^{3/2}$$

$$P_4^4(x) = 105(1-x^2)^2$$

$$P_5^0(x) = \frac{1}{8}(63x^5 - 70x^3 + 15x)$$

$$P_5^1(x) = \frac{15}{8}(21x^4 - 14x^2 + 1)(1-x^2)^{1/2}$$

$$P_5^2(x) = \frac{105}{2}(3x^3 - x)(1-x^2)$$

$$P_5^3(x) = \frac{105}{2}(9x^2 - 1)(1-x^2)^{3/2}$$

$$P_5^4(x) = 945x(1-x^2)^2$$

$$P_5^5(x) = 945(1-x^2)^{5/2}$$

$$P_6^0(x) = \frac{1}{16}(231x^6 - 315x^4 + 105x^2 - 5)$$

$$P_6^1(x) = \frac{21}{8}(33x^5 - 30x^3 + 5x)(1-x^2)^{1/2}$$

$$P_6^2(x) = \frac{105}{8}(33x^4 - 18x^2 + 1)(1-x^2)$$

$$P_6^3(x) = \frac{315}{2}(11x^3 - 3x)(1-x^2)^{3/2}$$

$$P_6^4(x) = \frac{945}{2}(11x^2 - 1)(1-x^2)^2$$

$$P_6^5(x) = 10395x(1-x^2)^{5/2}$$

$$P_6^6(x) = 10395(1-x^2)^3$$

$$P_7^0(x) = \frac{1}{16}(429x^7 - 693x^5 + 315x^3 - 35x)$$

$$P_7^1(x) = \frac{7}{16}(429x^6 - 495x^4 + 135x^2 - 5)(1-x^2)^{1/2}$$

$$P_7^2(x) = \frac{63}{8}(143x^5 - 110x^3 + 15x)(1-x^2)$$

$$P_7^3(x) = \frac{315}{8}(143x^4 - 66x^2 + 3)(1-x^2)^{3/2}$$

$$P_7^4(x) = \frac{3465}{2}(13x^3 - 3x)(1-x^2)^2$$

$$P_7^5(x) = \frac{10395}{2}(13x^2 - 1)(1-x^2)^{5/2}$$

$$P_7^6(x) = 135135x(1-x^2)^3$$

$$P_7^7(x) = 135135(1-x^2)^{7/2}$$

$$P_8^0(x) = \frac{1}{128}(6435x^8 - 12012x^6 + 6930x^4 - 1260x^2 + 35)$$

$$P_8^1(x) = \frac{9}{16}(715x^7 - 1001x^5 + 385x^3 - 35x)(1-x^2)^{1/2}$$

$$P_8^2(x) = \frac{315}{16}(143x^6 - 143x^4 + 33x^2 - 1)(1-x^2)$$

$$P_8^3(x) = \frac{3465}{8}(39x^5 - 26x^3 + 3x)(1-x^2)^{3/2}$$

$$P_8^4(x) = \frac{10395}{8}(65x^4 - 26x^2 + 1)(1-x^2)^2$$

$$P_8^5(x) = \frac{135135}{2}(5x^3 - x)(1-x^2)^{5/2}$$

$$P_8^6(x) = \frac{135135}{2}(15x^2 - 1)(1-x^2)^3$$

$$P_8^7(x) = 2027025x(1-x^2)^{7/2}$$

$$P_8^8(x) = 2027025(1-x^2)^4$$

$$P_9^0(x) = \frac{1}{128}(12155x^9 - 25740x^7 + 18018x^5 - 4620x^3 + 315x)$$

$$P_9^1(x) = \frac{45}{128}(2431x^8 - 4004x^6 + 2002x^4 - 308x^2 + 7)(1-x^2)^{1/2}$$

$$P_9^2(x) = \frac{495}{16}(221x^7 - 273x^5 + 91x^3 - 7x)(1-x^2)$$

$$P_9^3(x) = \frac{3465}{16}(221x^6 - 195x^4 + 39x^2 - 1)(1-x^2)^{3/2}$$

$$P_9^4(x) = \frac{135135}{8}(17x^5 - 10x^3 + x)(1-x^2)^2$$

$$P_9^5(x) = \frac{135135}{8}(85x^4 - 30x^2 + 1)(1-x^2)^{5/2}$$

$$P_9^6(x) = \frac{675675}{2}(17x^3 - 3x)(1-x^2)^3$$

$$P_9^7(x) = \frac{2027025}{2}(17x^2 - x)(1-x^2)^{7/2}$$

$$P_9^8(x) = 34459425x(1-x^2)^4$$

$$P_9^9(x) = 34459425(1-x^2)^{9/2}$$

$$P_{10}^0(x) = \frac{1}{256} (46189x^{10} - 109395x^8 + 90090x^6 - 30030x^4 + 3465x^2 - 63)$$

$$P_{10}^1(x) = \frac{55}{128} (4199x^9 - 7956x^7 + 4914x^5 - 1092x^3 + 63x)(1-x^2)^{1/2}$$

$$P_{10}^2(x) = \frac{495}{128} (4199x^8 - 6188x^6 + 2730x^4 - 364x^2 + 7)(1-x^2)$$

$$P_{10}^3(x) = \frac{6435}{16} (323x^7 - 357x^5 + 105x^3 - 7x)(1-x^2)^{3/2}$$

$$P_{10}^4(x) = \frac{45045}{16} (323x^6 - 255x^4 + 45x^2 - 1)(1-x^2)^2$$

$$P_{10}^5(x) = \frac{135135}{8} (323x^5 - 170x^3 + 15x)(1-x^2)^{5/2}$$

$$P_{10}^6(x) = \frac{675675}{8} (323x^4 - 102x^2 + 3)(1-x^2)^3$$

$$P_{10}^7(x) = \frac{11486475}{2} (19x^3 - 3x)(1-x^2)^{7/2}$$

$$P_{10}^8(x) = \frac{34459425}{2} (19x^2 - 1)(1-x^2)^4$$

$$P_{10}^9(x) = 654729075x(1-x^2)^{9/2}$$

$$P_{10}^{10}(x) = 654729075(1-x^2)^5$$