

Quantum Mechanics

Table of Spherical harmonics

$$Y_0^0 = \frac{1}{2} \sqrt{\frac{1}{\pi}}$$

$$Y_1^0 = \frac{1}{2} \sqrt{\frac{3}{\pi}} \cos \theta$$

$$Y_1^{\pm 1} = \mp \frac{1}{2} \sqrt{\frac{3}{2\pi}} \sin \theta e^{\pm \phi i}$$

$$Y_2^0 = \frac{1}{4} \sqrt{\frac{5}{\pi}} (3 \cos^2 \theta - 1)$$

$$Y_2^{\pm 1} = \mp \frac{1}{2} \sqrt{\frac{15}{2\pi}} \cos \theta \sin \theta e^{\pm \phi i}$$

$$Y_2^{\pm 2} = \frac{1}{4} \sqrt{\frac{15}{2\pi}} \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_3^0 = \frac{1}{4} \sqrt{\frac{7}{\pi}} (5 \cos^3 \theta - 3 \cos \theta)$$

$$Y_3^{\pm 1} = \mp \frac{1}{8} \sqrt{\frac{21}{\pi}} (5 \cos^2 \theta - 1) \sin \theta e^{\pm \phi i}$$

$$Y_3^{\pm 2} = \frac{1}{4} \sqrt{\frac{105}{2\pi}} \cos \theta \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_3^{\pm 3} = \mp \frac{1}{8} \sqrt{\frac{35}{\pi}} \sin^3 \theta e^{\pm 3\phi i}$$

$$Y_4^0 = \frac{3}{16} \sqrt{\frac{1}{\pi}} (35 \cos^4 \theta - 30 \cos^2 \theta + 3)$$

$$Y_4^{\pm 1} = \mp \frac{3}{8} \sqrt{\frac{5}{\pi}} (7 \cos^3 \theta - 3 \cos \theta) \sin \theta e^{\pm \phi i}$$

$$Y_4^{\pm 2} = \frac{3}{8} \sqrt{\frac{5}{2\pi}} (7 \cos^2 \theta - 1) \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_4^{\pm 3} = \mp \frac{3}{8} \sqrt{\frac{35}{\pi}} \cos \theta \sin^3 \theta e^{\pm 3\phi i}$$

$$Y_4^{\pm 4} = \frac{3}{16} \sqrt{\frac{35}{2\pi}} \sin^4 \theta e^{\pm 4\phi i}$$

$$Y_5^0 = \frac{1}{16} \sqrt{\frac{11}{\pi}} (63 \cos^5 \theta - 70 \cos^3 \theta + 15 \cos \theta)$$

$$Y_5^{\pm 1} = \mp \frac{1}{16} \sqrt{\frac{165}{2\pi}} (21 \cos^4 \theta - 14 \cos^2 \theta + 1) \sin \theta e^{\pm \phi i}$$

$$Y_5^{\pm 2} = \frac{1}{8} \sqrt{\frac{1155}{2\pi}} (3 \cos^3 \theta - \cos \theta) \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_5^{\pm 3} = \mp \frac{1}{32} \sqrt{\frac{385}{\pi}} (9 \cos^2 \theta - 1) \sin^3 \theta e^{\pm 3\phi i}$$

$$Y_5^{\pm 4} = \frac{3}{16} \sqrt{\frac{385}{2\pi}} \cos \theta \sin^4 \theta e^{\pm 4\phi i}$$

$$Y_5^{\pm 5} = \mp \frac{3}{32} \sqrt{\frac{77}{\pi}} \sin^5 \theta e^{\pm 5\phi i}$$

$$Y_6^0 = \frac{1}{32} \sqrt{\frac{13}{\pi}} (231 \cos^6 \theta - 315 \cos^4 \theta + 105 \cos^2 \theta - 5)$$

$$Y_6^{\pm 1} = \mp \frac{1}{16} \sqrt{\frac{273}{2\pi}} (33 \cos^5 \theta - 30 \cos^3 \theta + 5 \cos \theta) \sin \theta e^{\pm \phi i}$$

$$Y_6^{\pm 2} = \frac{1}{64} \sqrt{\frac{1365}{\pi}} (33 \cos^4 \theta - 18 \cos^2 \theta + 1) \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_6^{\pm 3} = \mp \frac{1}{32} \sqrt{\frac{1365}{\pi}} (11 \cos^3 \theta - 3x) \sin^3 \theta e^{\pm 3\phi i}$$

$$Y_6^{\pm 4} = \frac{3}{32} \sqrt{\frac{91}{2\pi}} (11 \cos^2 \theta - 1) \sin^4 \theta e^{\pm 4\phi i}$$

$$Y_6^{\pm 5} = \mp \frac{3}{32} \sqrt{\frac{1001}{\pi}} \cos \theta \sin^5 \theta e^{\pm 5\phi i}$$

$$Y_6^{\pm 6} = \frac{1}{64} \sqrt{\frac{3003}{\pi}} \sin^6 \theta e^{\pm 6\phi i}$$

$$Y_7^0 = \frac{1}{32} \sqrt{\frac{15}{\pi}} (429 \cos^7 \theta - 693 \cos^5 \theta + 315 \cos^3 \theta - 35 \cos \theta)$$

$$Y_7^{\pm 1} = \mp \frac{1}{64} \sqrt{\frac{105}{2\pi}} (429 \cos^6 \theta - 495 \cos^4 \theta + 135 \cos^2 \theta - 5) \sin \theta e^{\pm \phi i}$$

$$Y_7^{\pm 2} = \frac{3}{64} \sqrt{\frac{35}{\pi}} (143 \cos^5 \theta - 110 \cos^3 \theta + 15 \cos \theta) \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_7^{\pm 3} = \mp \frac{3}{64} \sqrt{\frac{35}{2\pi}} (143 \cos^4 \theta - 66 \cos^2 \theta + 3) \sin^3 \theta e^{\pm 3\phi i}$$

$$Y_7^{\pm 4} = \frac{3}{32} \sqrt{\frac{385}{2\pi}} (13 \cos^3 \theta - 3 \cos \theta) \sin^4 \theta e^{\pm 4\phi i}$$

$$Y_7^{\pm 5} = \mp \frac{3}{64} \sqrt{\frac{385}{2\pi}} (13 \cos^2 \theta - 1) \sin^5 \theta e^{\pm 5\phi i}$$

$$Y_7^{\pm 6} = \frac{3}{64} \sqrt{\frac{5005}{\pi}} \cos \theta \sin^6 \theta e^{\pm 6\phi i}$$

$$Y_7^{\pm 7} = \mp \frac{3}{64} \sqrt{\frac{715}{2\pi}} \sin^7 \theta e^{\pm 7\phi i}$$

$$Y_8^0 = \frac{1}{256} \sqrt{\frac{17}{\pi}} (6435 \cos^8 \theta - 12012 \cos^6 \theta + 6930 \cos^4 \theta - 1260 \cos^2 \theta + 35)$$

$$Y_8^{\pm 1} = \mp \frac{3}{64} \sqrt{\frac{17}{2\pi}} (715 \cos^7 \theta - 1001 \cos^5 \theta + 385 \cos^3 \theta - 35 \cos \theta) \sin \theta e^{\pm \phi i}$$

$$Y_8^{\pm 2} = \frac{3}{128} \sqrt{\frac{595}{\pi}} (143 \cos^6 \theta - 143 \cos^4 \theta + 33 \cos^2 \theta - 1) \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_8^{\pm 3} = \mp \frac{1}{64} \sqrt{\frac{19635}{2\pi}} (39 \cos^5 \theta - 26 \cos^3 \theta + 3 \cos \theta) \sin^3 \theta e^{\pm 3\phi i}$$

$$Y_8^{\pm 4} = \frac{3}{128} \sqrt{\frac{1309}{2\pi}} (65 \cos^4 \theta - 26 \cos^2 \theta + 1) \sin^4 \theta e^{\pm 4\phi i}$$

$$Y_8^{\pm 5} = \mp \frac{3}{64} \sqrt{\frac{17017}{2\pi}} (5 \cos^3 \theta - x) \sin^5 \theta e^{\pm 5\phi i}$$

$$Y_8^{\pm 6} = \frac{1}{128} \sqrt{\frac{7293}{\pi}} (15 \cos^2 \theta - 1) \sin^6 \theta e^{\pm 6\phi i}$$

$$Y_8^{\pm 7} = \mp \frac{3}{64} \sqrt{\frac{12155}{2\pi}} \cos \theta \sin^7 \theta e^{\pm 7\phi i}$$

$$Y_8^{\pm 8} = \frac{3}{256} \sqrt{\frac{12155}{2\pi}} \sin^8 \theta e^{\pm 8\phi i}$$

$$Y_9^0 = \frac{1}{256} \sqrt{\frac{19}{\pi}} (12155 \cos^9 \theta - 25740 \cos^7 \theta + 18018 \cos^5 \theta - 4620 \cos^3 \theta + 315 \cos \theta)$$

$$Y_9^{\pm 1} = \mp \frac{3}{256} \sqrt{\frac{95}{2\pi}} (2431 \cos^8 \theta - 4004 \cos^6 \theta + 2002 \cos^4 \theta - 308 \cos^2 \theta + 7) \sin \theta e^{\pm \phi i}$$

$$Y_9^{\pm 2} = \frac{3}{128} \sqrt{\frac{1045}{\pi}} (221 \cos^7 \theta - 273 \cos^5 \theta + 91 \cos^3 \theta - 7 \cos \theta) \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_9^{\pm 3} = \mp \frac{1}{256} \sqrt{\frac{21945}{\pi}} (221 \cos^6 \theta - 195 \cos^4 \theta + 39 \cos^2 \theta - 1) \sin^3 \theta e^{\pm 3\phi i}$$

$$Y_9^{\pm 4} = \frac{3}{128} \sqrt{\frac{95095}{2\pi}} (17 \cos^5 \theta - 10 \cos^3 \theta + \cos \theta) \sin^4 \theta e^{\pm 4\phi i}$$

$$Y_9^{\pm 5} = \mp \frac{3}{256} \sqrt{\frac{2717}{\pi}} (85 \cos^4 \theta - 30 \cos^2 \theta + 1) \sin^5 \theta e^{\pm 5\phi i}$$

$$Y_9^{\pm 6} = \frac{1}{128} \sqrt{\frac{40755}{\pi}} (17 \cos^3 \theta - 3 \cos \theta) \sin^6 \theta e^{\pm 6\phi i}$$

$$Y_9^{\pm 7} = \mp \frac{3}{512} \sqrt{\frac{13585}{\pi}} (17 \cos^2 \theta - \cos \theta) \sin^7 \theta e^{\pm 7\phi i}$$

$$Y_9^{\pm 8} = \frac{3}{256} \sqrt{\frac{230945}{2\pi}} \cos \theta \sin^8 \theta e^{\pm 8\phi i}$$

$$Y_9^{\pm 9} = \mp \frac{1}{512} \sqrt{\frac{230945}{\pi}} \sin^9 \theta e^{\pm 9\phi i}$$

$$Y_{10}^0 = \frac{1}{512} \sqrt{\frac{21}{\pi}} (46189 \cos^{10} \theta - 109395 \cos^8 \theta + 90090 \cos^6 \theta - 30030 \cos^4 \theta + 3465 \cos^2 \theta - 63)$$

$$Y_{10}^{\pm 1} = \mp \frac{1}{256} \sqrt{\frac{1155}{2\pi}} (4199 \cos^9 \theta - 7956 \cos^7 \theta + 4914 \cos^5 \theta - 1092 \cos^3 \theta + 63 \cos \theta) \sin \theta e^{\pm \phi i}$$

$$Y_{10}^{\pm 2} = \frac{3}{512} \sqrt{\frac{385}{2\pi}} (4199 \cos^8 \theta - 6188 \cos^6 \theta + 2730 \cos^4 \theta - 364 \cos^2 \theta + 7) \sin^2 \theta e^{\pm 2\phi i}$$

$$Y_{10}^{\pm 3} = \mp \frac{3}{256} \sqrt{\frac{5005}{\pi}} (323 \cos^7 \theta - 357 \cos^5 \theta + 105 \cos^3 \theta - 7 \cos \theta) \sin^3 \theta e^{\pm 3\phi i}$$

$$Y_{10}^{\pm 4} = \frac{3}{256} \sqrt{\frac{5005}{2\pi}} (323 \cos^6 \theta - 255 \cos^4 \theta + 45 \cos^2 \theta - 1) \sin^4 \theta e^{\pm 4\phi i}$$

$$Y_{10}^{\pm 5} = \mp \frac{3}{256} \sqrt{\frac{1001}{\pi}} (323 \cos^5 \theta - 170 \cos^3 \theta + 15 \cos \theta) \sin^5 \theta e^{\pm 5\phi i}$$

$$Y_{10}^{\pm 6} = \frac{3}{1024} \sqrt{\frac{5005}{\pi}} (323 \cos^4 \theta - 102 \cos^2 \theta + 3) \sin^6 \theta e^{\pm 6\phi i}$$

$$Y_{10}^{\pm 7} = \mp \frac{3}{512} \sqrt{\frac{85085}{\pi}} (19 \cos^3 \theta - 3 \cos \theta) \sin^7 \theta e^{\pm 7\phi i}$$

$$Y_{10}^{\pm 8} = \frac{1}{512} \sqrt{\frac{255255}{2\pi}} (19 \cos^2 \theta - 1) \sin^8 \theta e^{\pm 8\phi i}$$

$$Y_{10}^{\pm 9} = \mp \frac{1}{512} \sqrt{\frac{4849845}{\pi}} \cos \theta \sin^9 \theta e^{\pm 9\phi i}$$

$$Y_{10}^{\pm 10} = \frac{1}{1024} \sqrt{\frac{969969}{\pi}} \sin^{10} \theta e^{\pm 10\phi i}$$